Engineering Notes

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Burning Rate Temperature Sensitivity of Composite Solid Propellants

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THE purpose of this investigation was to obtain a relationship between propellant burning rate and initial propellant temperature which explicitly contains those physical parameters most influential in affecting burning-rate temperature sensitivity. Of the combustion models for composite solid propellants, the granular diffusion flame (GDF) model was selected since it provides a more compact qualitative and quantitative representation of the combustion processes than any other.[1–3]

In the analytical study the physical properties included in the GDF burning rate equation were written in terms of the initial temperature T_0, and the resulting equation was expanded term by term using the binomial theorem. With property values typical of an AP-based composite propellant, an order-of-magnitude analysis of the expanded terms leads to a burning rate expression which explicitly includes T_0:

$$r^{-1} = (1 - \Delta T_0/u)^{1/2}(a'/p + b'/p^{1/3}) \quad (1)$$

where p = pressure, a' and b' are constants, and

$$u = T_s - T_0' - Q_s/C_s; \quad \Delta T_0 = T_0 - T_0' \quad (2)$$

Since $(a'/p + b'/p^{1/3}) = r^{-1}$ at some reference temperature T_0', Eq. (1) becomes

$$r'/r = (1 - \Delta T_0/u)^{1/2} \quad (3)$$

The burning rate temperature sensitivity σ_p is obtained by differentiating Eq. (3) with respect to initial temperature at constant pressure

$$\sigma_p = 1/2(u - \Delta T_0) \quad (4)$$

The parameters significant in affecting r and σ_p occur naturally in the temperature effects parameter u; they are propellant burning surface temperature T_s, the surface heat release, Q_s, and the specific heat of the solid propellant, C_s. The importance of these same parameters on σ_p was the first suggested by Glick.[4] Since T_s, Q_s, and C_s, are assumed to be constants in the GDF model, u is also constant.

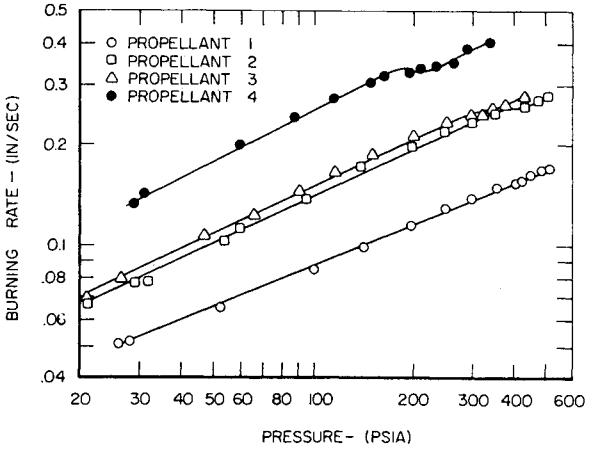

Fig. 1 Data for propellants T_0 = 70°F.

Experimental Results

Strands of four composite-propellant formulations (Table 1) were burned in a modified Crawford type strand burner. The binder is (PBAA); the oxidizer is ammonium perchlorate (AP); and the catalyst is copper chromite.

All tests were conducted in the pressure range from 20 to 500 psia where the surface heat release Q_s, was expected to be most significant in influencing the burning rate.

Figure 1 presents log-log plots of r vs p at 70°F. Slope changes occur in the curves for all propellants. This "intermittent burning" is explained by Summerfield, et al., in terms of the condition of the binder at the burning surface.[3] If the binder is molten and sufficiently fluid to cover the oxidizer particles, then the propellant flame is locally extinguished and burning may be intermittent. The PBAA binder used here is classified as one which supposedly does not readily melt.

The burning-rate equation obtained from the GDF model cannot predict the pressure at which intermittent burning will occur. However, the model does include the basic mechanism of combustion up to that pressure. Since the GDF model is the basis for the efforts undertaken, it was applied only to burning-rate data collected at pressures below the onset of intermittent burning. If the GDF model applies for these conditions, then the data must be correlated by a straight line whose equation is

$$p/r = a + bp^{2/3} \quad (5)$$

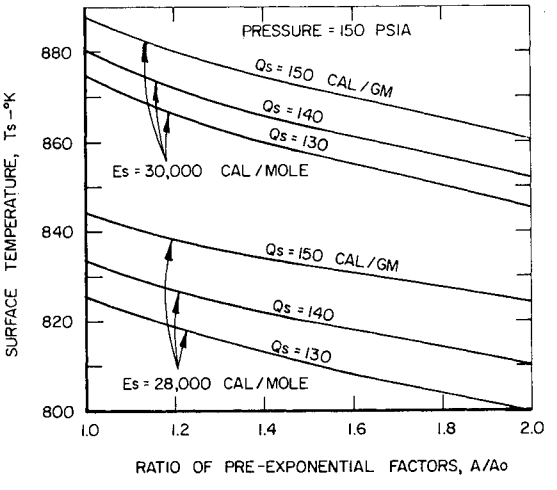

Fig. 2 Effect of pre-exponential factor, activation energy, and surface heat release on propellant surface temperature.

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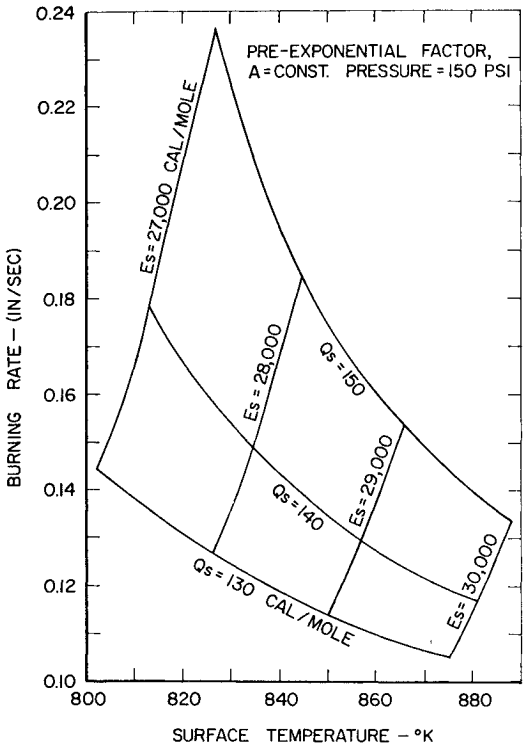

Fig. 3 Effect of activation energy and surface heat release on propellant surface temperature.

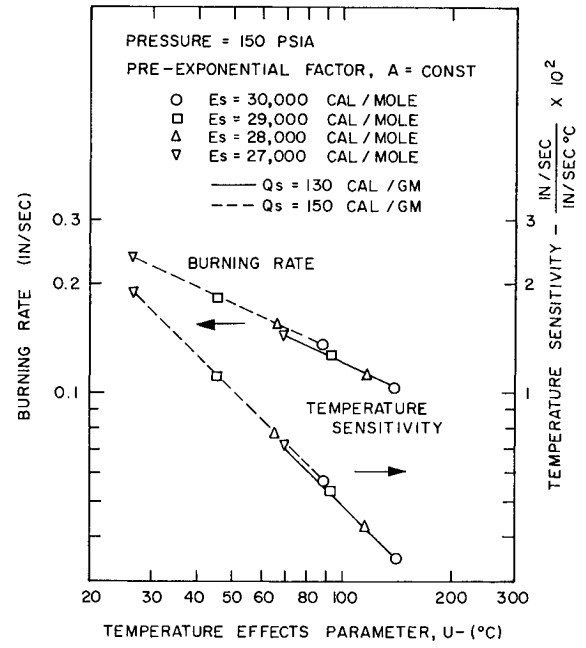

Fig. 4 Effect of activation energy and surface heat release on burning rate, temperature effects parameter, and temperature sensitivity.

A weighted regression analysis was performed on the measured data for each propellant. This analysis indicated that a linear relationship of the form of Eq. (5) will correlate the data. The constants, determined by the method of least squares, are given in Table 1 as are values of u.

Discussion

Since the parameter u evolves directly from the GDF model, it should be determined whether u could be represented by a constant for a particular propellant formulation. Within the limits of experimental error, the values of u tabulated in Table 1 are constants for each propellant. Using the average value of u, Eq. (1) will accurately correlate r, p, and T_0 for the propellants tested. At this point, there is no substantiated experimental evidence to indicate that u is anything other than an empirical constant chosen to correlate the experimental data. Thus, significant formulational changes were selected with the intention of showing that the temperature effects parameter is in fact a grouping of physically significant propellant combustion properties, T_s, C_s, and Q_s.

Catalytic Effect

Certain recent experimental work indicates that a burning rate catalyst, such as copper chromite, acts to increase r in several ways. At high pressures, 125–500 psia, it reduces the activation energy of the over-all solid to gas decomposition step, while at low pressures, 15–125 psia, it appears to increase the pre-exponential factor in the Arrhenius expression for the overall surface decomposition.[5] The location of primary catalytic action is believed to be in the gas phase reaction between ammonia (A) and perchloric acid (PA) immediately above the AP surface.[6–8] Other experiments[5] indicate that the catalyst promotes heterogeneous surface reactions. Since Q_s in Eq. (2) includes the heat release from the A-PA reaction as well as the heat release from heterogeneous reactions, the surface heat release should increase with the addition of copper chromite.

The observed effects of the catalyst were incorporated into a parametric study of the GDF model for the collapsed A-PA reaction zone. The results of the parametric study are shown in Figs. 2 and 3 for propellant parameters typical of the propellants tested. The results indicate that the addition of copper chromite to an uncatalyzed propellant should decrease T_s and, hence, u.

Oxidizer Particle Size Effect

Since the GDF model assumes that T_s is independent of oxidizer particle size, and since a reduction in particle size exposes more surface area at which heterogeneous reactions might occur, a reduction in oxidizer particle size would be expected to reduce u slightly. However, u increased sharply for propellant 4. This increase can occur only if Q_s is greatly reduced or T_s is greatly increased. From previous considerations Q_s should not decrease; however, a drastic increase in T_s is conceivable. Reducing the particle size reduces the amount of interstitial fuel between adjacent oxidizer par-

Table 1 Propellant data[a]

Propellant No.	Composition, weight %			AP size, μ	Constants in Eq. (5)						u, °F		σ_p, at u_{avg} °F^{-1}
					T_0 = 70°F		T_0 = 100°F		T_0 = 130°F				
	Binder	AP	Catalyst		a	b	a	b	a	b	u_{70-100}	u_{70-130}	
1	30.0	70.0	0.0	80	158	45.1	145	42.9	130	40.7	312	323	0.00194
2	29.4	69.4	1.2	80	149	25.1	141	23.8	129	22.3	297	283	0.00219
3	28.8	68.8	2.4	80	126	24.4	123	22.9	122	21.0	250	232	0.00276
4	28.8	68.8	2.4	15	84	14.2	78	14.2	72	13.9	1154	1277	...

[a] $u_{avg} = (u_{70-100} + u_{70-130})/2$.

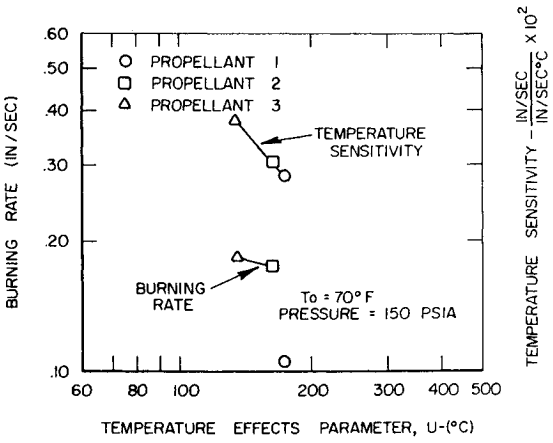

Fig. 5 Burning rate and temperature sensitivity vs temperature effects parameter-experimental data.

ticles and on the average exposes more oxidizer surface to the hot combustion gases. In this case the rapid exothermic A-PA reaction occurring very close to the AP particle, would become a more dominant influence in determining T_s, since this temperature is really an average gas phase temperature near the surface.

Over-All Effects

The final objective was to determine the effect of u on r and σ_p. The effects predicted by the parametric study are shown in Fig. 4. Here it appears that $r \propto u^{-1/2}$, and $\sigma_p \propto u^{-1}$. Figure 5 presents experimental data. For propellants 2 and 3, r follows the trend predicted in Fig. 4. The initial increase in r with the catalyst addition, i.e., the increase from uncatalyzed propellant 1 to propellant 2, does not fit the predicted trend. This merely means that the increase in r is not solely due to changes in u. This condition is also apparent from the data of Table 1 where the initial addition of copper chromite reduces the diffusion time parameter b by a much greater amount than the reaction time parameter a. Figure 5 clearly indicates that the catalyzed increase in r is not produced solely by changes in T_s and Q_s.

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Rendezvous Equations in the Vicinity of the Second Libration Point

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Nomenclature

B	=	gravitational coefficient
D	=	Earth-moon distance
f	=	control force
G,K	=	gravitational constant
L_2	=	second-Lagrange point
M_e,M_m	=	masses of the Earth and moon
POI	=	point of injection
r_e,r_m	=	radial distances from Earth and moon
T	=	time at which rendezvous takes place
TPI	=	terminal phase initiation
ΔV_{1_i}	=	TPI ΔV maneuver
$\Delta V_{2i},\Delta V_{2_o}$	=	POI ΔV injection and braking maneuvers
ΔV_{2_A}	=	actual POI ΔV maneuver
μ	=	gravitational constant (0.01215)
ω	=	magnitude of angular velocity
ω_1,ω_2	=	magnitudes of the real and imaginary roots of the characteristic equation from the coupled equations (2.15863 and 1.86265, respectively)
ω_3	=	$K^{1/2}(K = 3.19044)$
ξ	=	distance from smaller body to the infinitesimal mass
r_t	=	magnitude of the position vector of the target

Subscripts

e,m	=	Earth and moon terms
o,o_d	=	initial and desired initial values

SINCE the conception of the Apollo Program there has been some concern over the loss of communications with the spacecraft as it goes behind the moon. General Electric[1] studied the possibility of using a lunar libration point relay satellite. They compared two schemes, a "stationary" or hummingbird lunar libration satellite which makes no motion relative to the Earth-moon system, and a satellite in a Halo orbit which orbits the second Lagrange point, while remaining continuously visible from the Earth. They concluded that the Halo orbit scheme was superior, due to its ability to maintain continuous communication with the Earth and its lesser stationkeeping requirements. Godfrey, Coffman, and Burr[2] carried the comparison further by considering an 8000-naut mile lunar orbiter scheme involving 2–3 satellites. They also chose the Halo scheme. This Note provides a simplified approach to the calculation of the insertion ΔV needed to inject the spacecraft into a Halo orbit about the second Lagrange point. It is based on "Project Linus," conducted by senior aerospace engineering students.[3]

The typical rendezvous problem considers a target and rendezvous vehicle in orbit about a celestial body—after a specified time the rendezvous docking occurs. One constraint is placed on the problem. The rendezvous vehicle must be relatively close to the target vehicle in order to linearize the equations of motion.

Now consider the restricted circular three body problem involving the Earth, moon, and a spacecraft. The L_2 point is one of five Lagrange points in a rotating coordinate system. A satellite placed at rest at one of these points will tend to remain there since the centripetal and gravitational accelerations acting on the satellite will be in equilibrium.[4] The L_2 point is one of the three colinear points on the Earth-moon line, and is 34,775 naut miles beyond the moon.

By considering the L_2 point as the target vehicle, and the actual spacecraft as the rendezvous vehicle, one can evaluate

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